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THE SELECTION OF WEIGHTING FUNCTIONS FOR LINEAR ARRAYS USING DIFFERENT TECHNIQUES

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OUTLINE

- Linear Arrays and Weighting Functions
- Spectral Weighting Using Windows
- Array polynomials from the z-transform
- Minimum Beamwidth for a Specified Sidelobe Level
- Least Squares Error Pattern Synthesis
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- Pattern Sampling in the Wavenumber space
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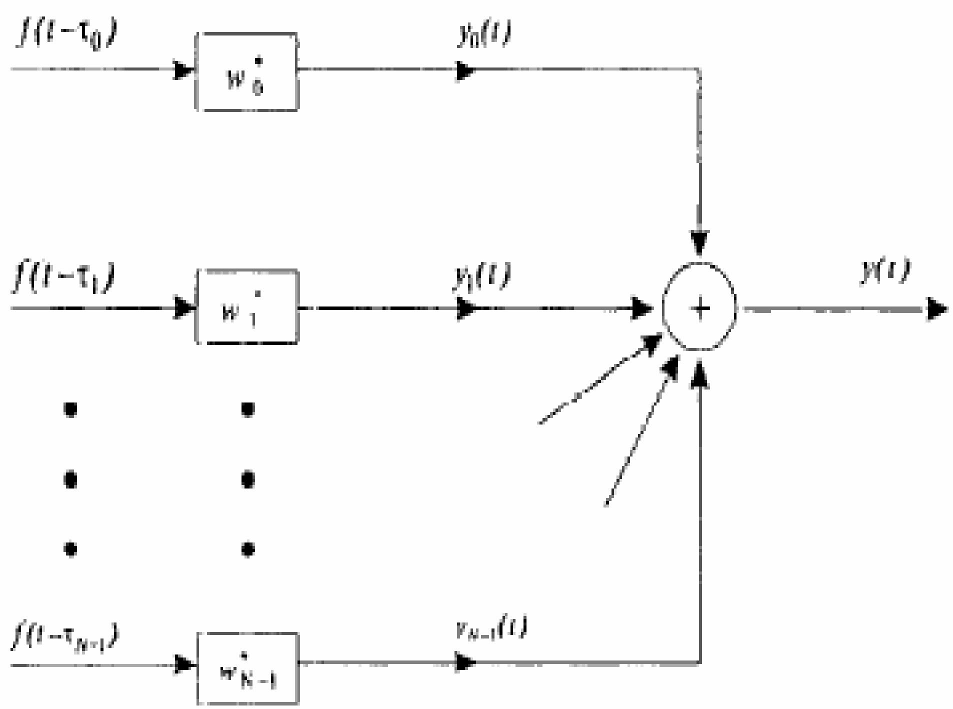
Linear Arrays and Weighting Functions

- A linear array is a one-dimensional array of antenna sources
- A narrowband beamformer is considered
- The beampattern of the array depends on the weights of the elements
- The input to each sensor is a delayed version of the incoming signal $f(t)$
- The corresponding sensor outputs are summed to form $y(t)$

A Narrowband Beamformer



$f(t)$	Incoming signal
τ_n	Delay of nth sensor
w_n	Weight of nth sensor
$y(t)$	Output signal
N	Number of sensors

Spectral Weighting Using Windows

- The weights w (n) can be chosen from different windows used in FIR filters:
- (a) Uniform
- (b) Hann
- (c) Hamming
- (d) Blackman-Harris
- (e) Raised Cosine
- We present a modified Hamming window

Hamming and Modified Hamming

- Hamming window (non-causal)

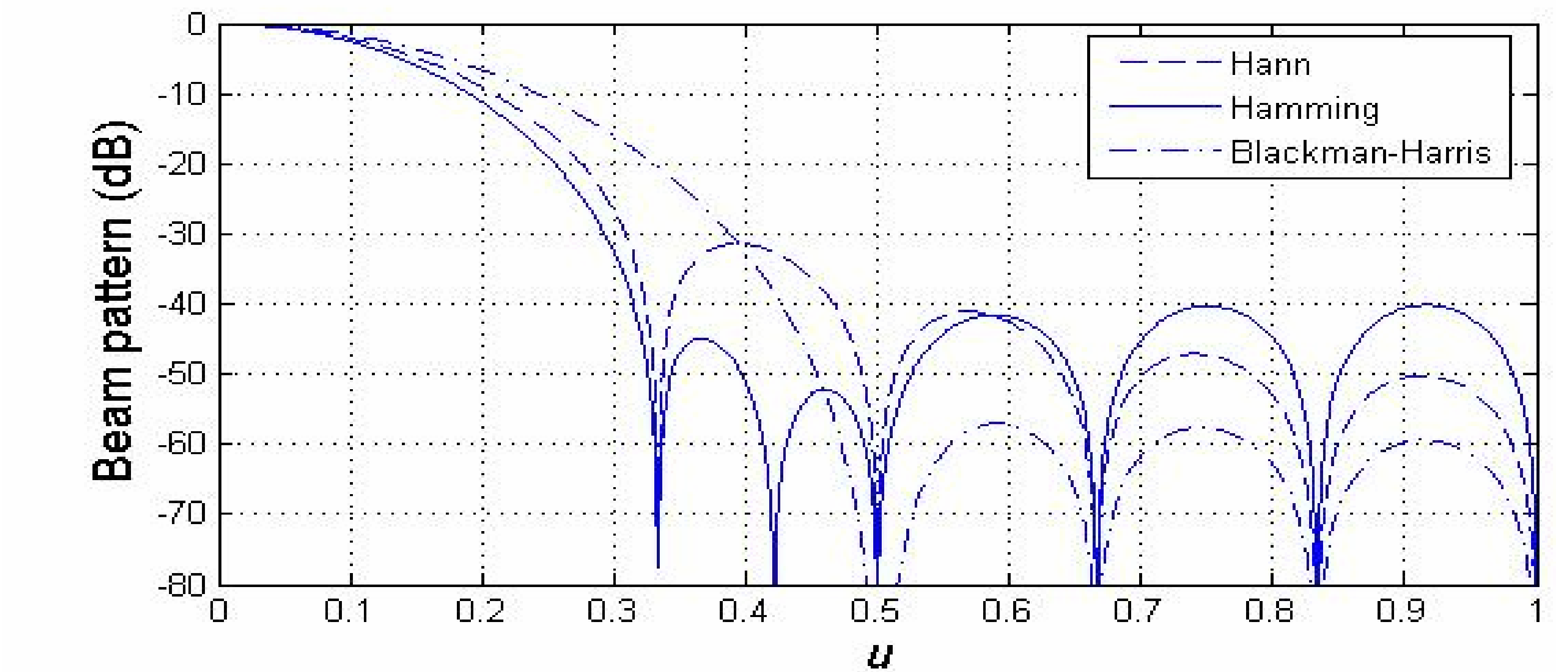
$$w(n) = 0.54 + 0.46\,Cos\left(\frac{2\pi n}{N}\right)$$

- Modified Hamming window (non-causal)

$$w(n) = 0.54 + 0.46\,Cos\left(\frac{4\pi n}{N}\right)\exp\left(\frac{-a^2 n^2}{N}\right)$$

- In both cases |n| ≤ N/2 , N assumed even

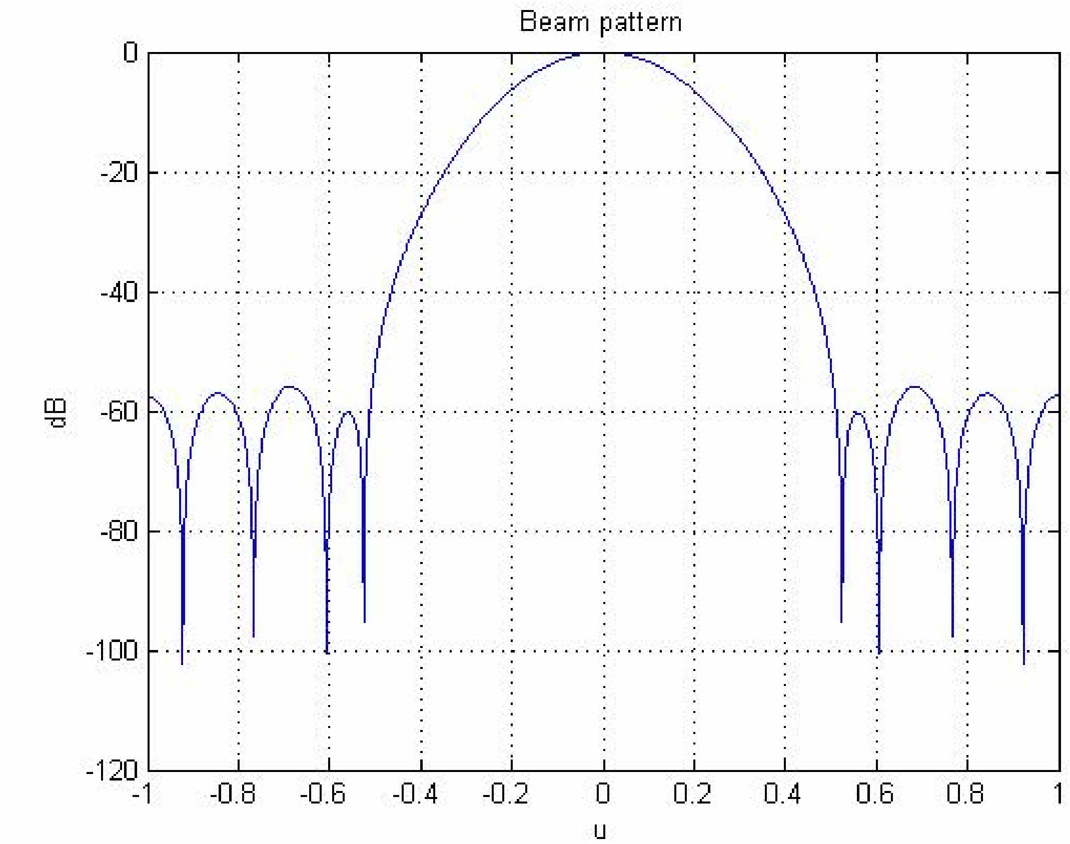
Beampattern – Other Windows



- u : unit vector along the single-dimension (x-axis)
- B (u) : beampattern of array. Here N = 12, d = λ / 2

Modified Hamming Window

- Beampattern $$B(u) = \sum_{n=-N/2}^{N/2} w(n) \exp(-j\, 2\pi\, dn\, u / \lambda)$$



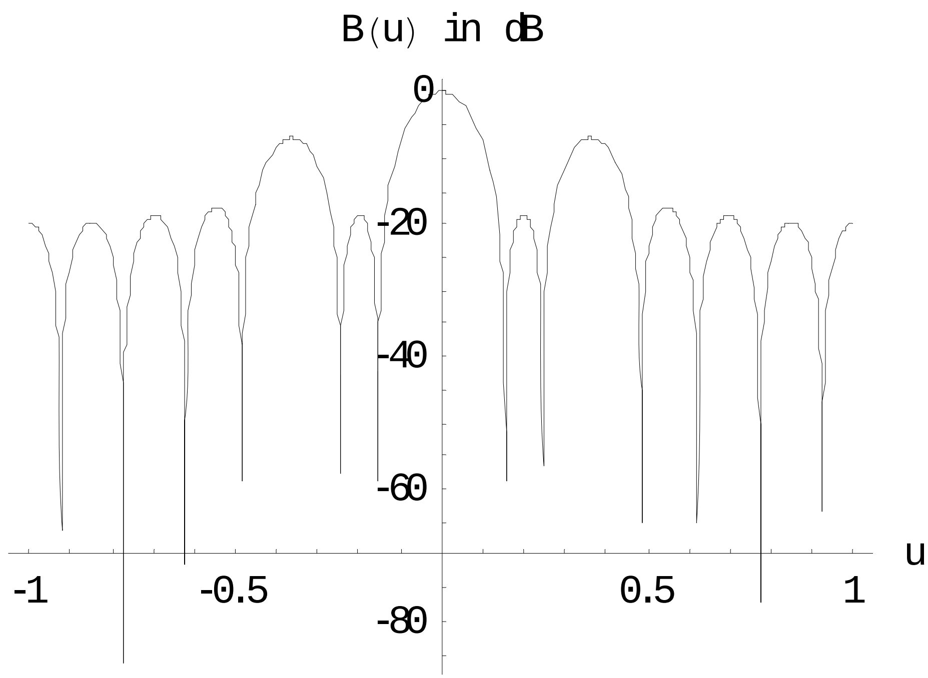
Effect on the Beampattern

- The negative exponential term helps in faster decay of the beampattern
- This provides a very less sidelobe level
- Hamming window : -45.0 dB
- Modified Hamming window : -60.3 dB
- The second harmonic cosine term increases First Null Beam-Width (FNBW)
- Hamming window : 0.70
- Modified Hamming window : 1.05

Flexibility of the Modified Window

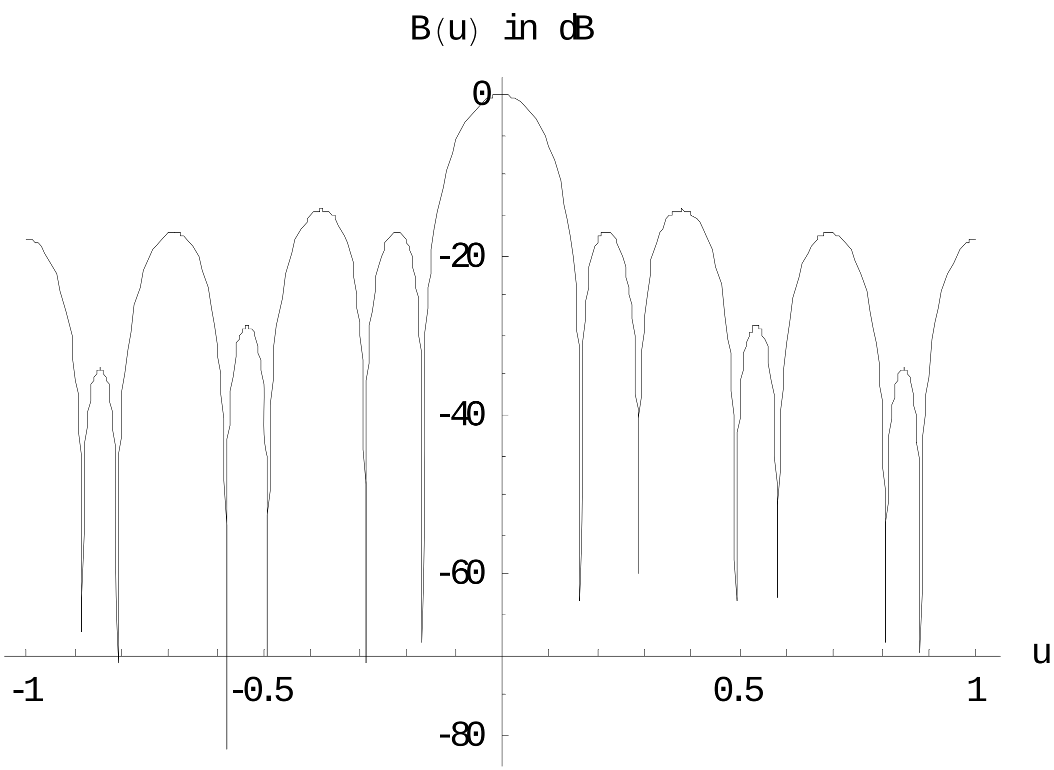
- The pattern shown in the previous figure corresponds to a = sqrt(3)
- We can choose the value of ‘a’ to obtain the desired sidelobe level, compensated by a small increase in beamwidth.
- Smaller values of ‘a’ cause a smaller peak sidelobe level compared to larger values of ‘a’.
- After considerable experimentation we found that the stable values of ‘a’ lie in the range 1.1 and 2.3

Modified Hamming a = 0.5



- The second sidelobe level is larger than first sidelobe.
- There is a gradual decay in all the lobes coming after the second sidelobe
- There is a considerable difference first sidelobe and second sidelobe

Modified Hamming a = 5.0



- The sidelobes 1,3,5,.. (odd) are smaller than 2,4,6… (even) sidelobes.

- The difference between first sidelobe and second sidelobe levels has comparatively reduced.

Array Polynomials from the Z-transform

- Let $B_z(z)$ be the unilateral Z-transform of weights. The beam pattern for *N* odd,

$$B_\psi(\psi) = [z^{-\frac{N-1}{2}} B_z(z)]|_{z=\exp(j\psi)}$$

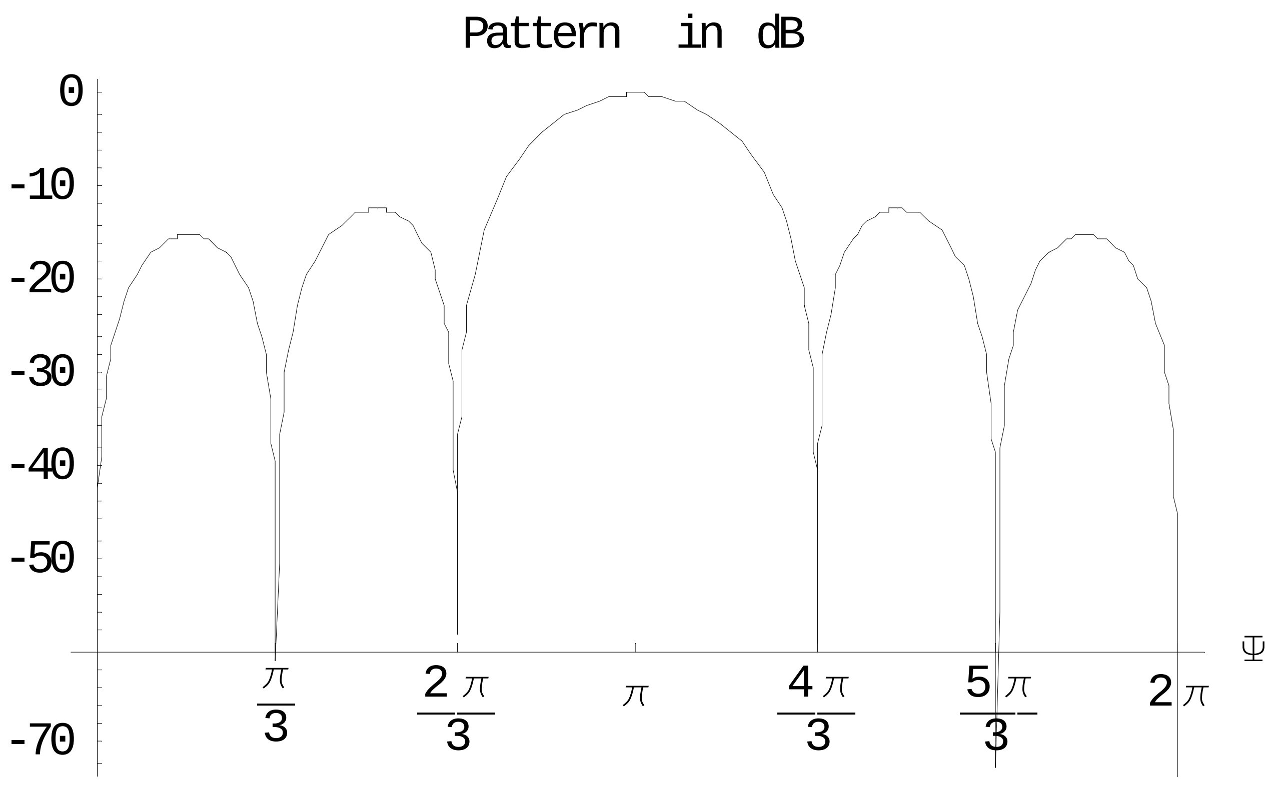
$$\psi = 2\pi d u / \lambda$$

- To place the nulls in the direction of 0° , 60° and 120°, the array polynomial must be

$$B_z(z) = (z-1)(z-e^{j\pi/3})(z-e^{-j\pi/3})(z-e^{j2\pi/3})(z-e^{-j2\pi/3})$$

$$= -1 + z - z^2 + z^3 - z^4 + z^5$$

An Example



Minimum Beamwidth for a Specified Sidelobe Level

- Sidelobe Ratio

$$R = \frac{\text{mainlobe maximum}}{\text{sidelobe level}}$$

- We find x_0 such that

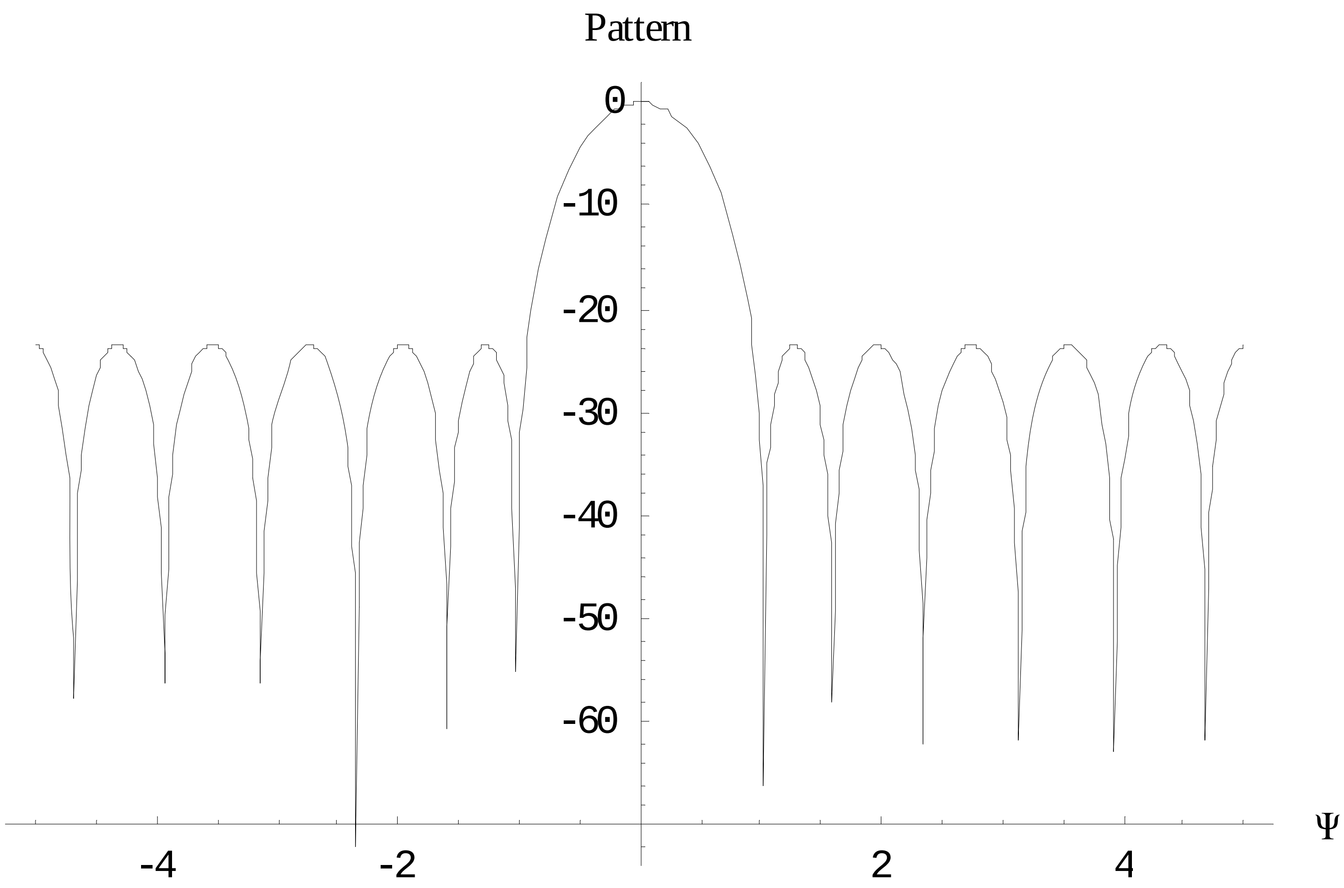
$$T_{N-1}(x_0) = R$$

- Beam pattern

$$B(\psi) = \frac{1}{R} T_{N-1}\left(x_0 \cos\left(\frac{\psi}{2}\right)\right)$$

- $T_n(x)$ the nth order Chebyshev polynomial of first kind
- Important Feature : All the sidelobes have the **SAME** minimum sidelobe level

Beam pattern – Dolph-Chebyshev Method

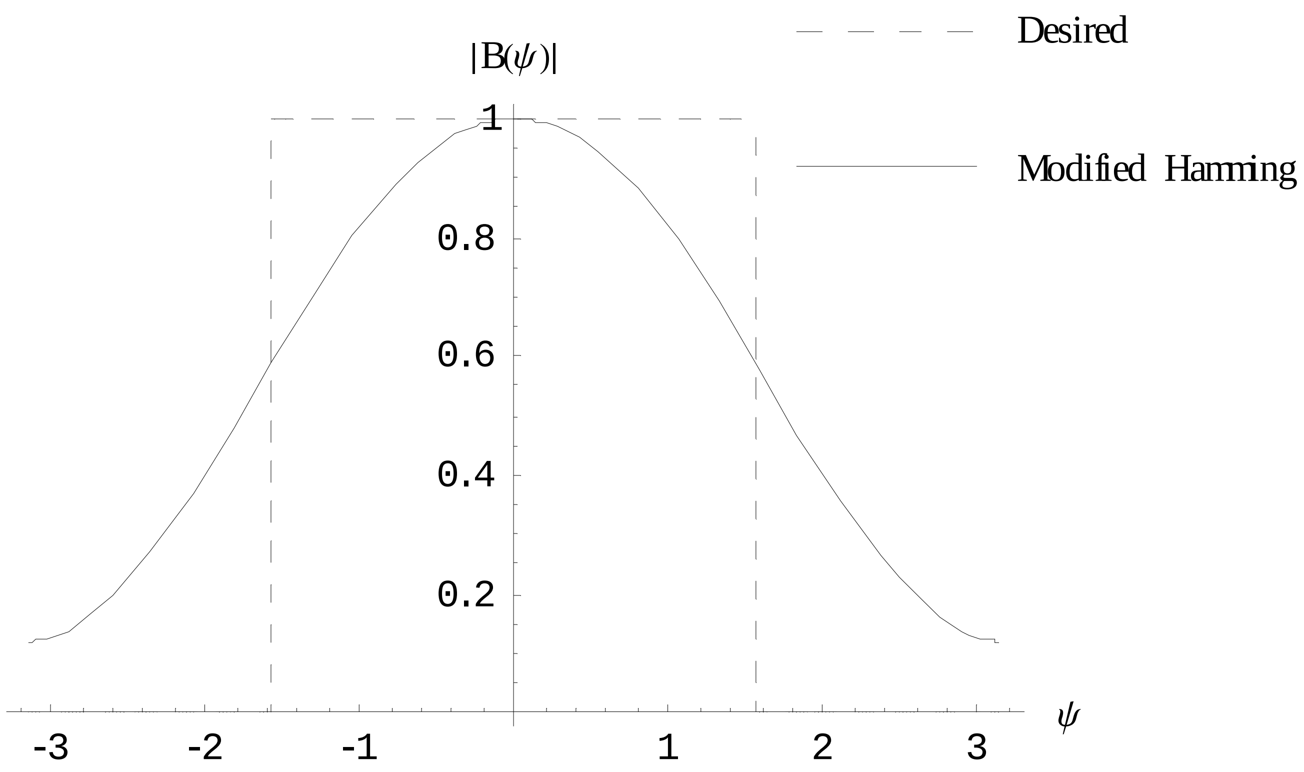


Least Squares Error Pattern Synthesis

- We minimize the square error between ideal beampattern and the windowed beampattern
- This characterization leads to a Fourier Series relationship between the weights and beampattern.
- The Fourier coefficients are multiplied by a window function for spectral smoothing.

$$\hat{B}(\psi) = \sum_{k=-N/2}^{N/2} \frac{\sin((k-0.5)\pi/2)}{(k-0.5)\pi/2} (0.54 + 0.46\cos(4\pi k/N)) \exp(-a^2 k^2/N)\, e^{jk\psi}$$

Least Squares Error Pattern Synthesis



Null Steering

- We intentionally want to place nulls in a particular direction.
- To place *M* nulls in the direction u_i ,(i=1,2,..M)

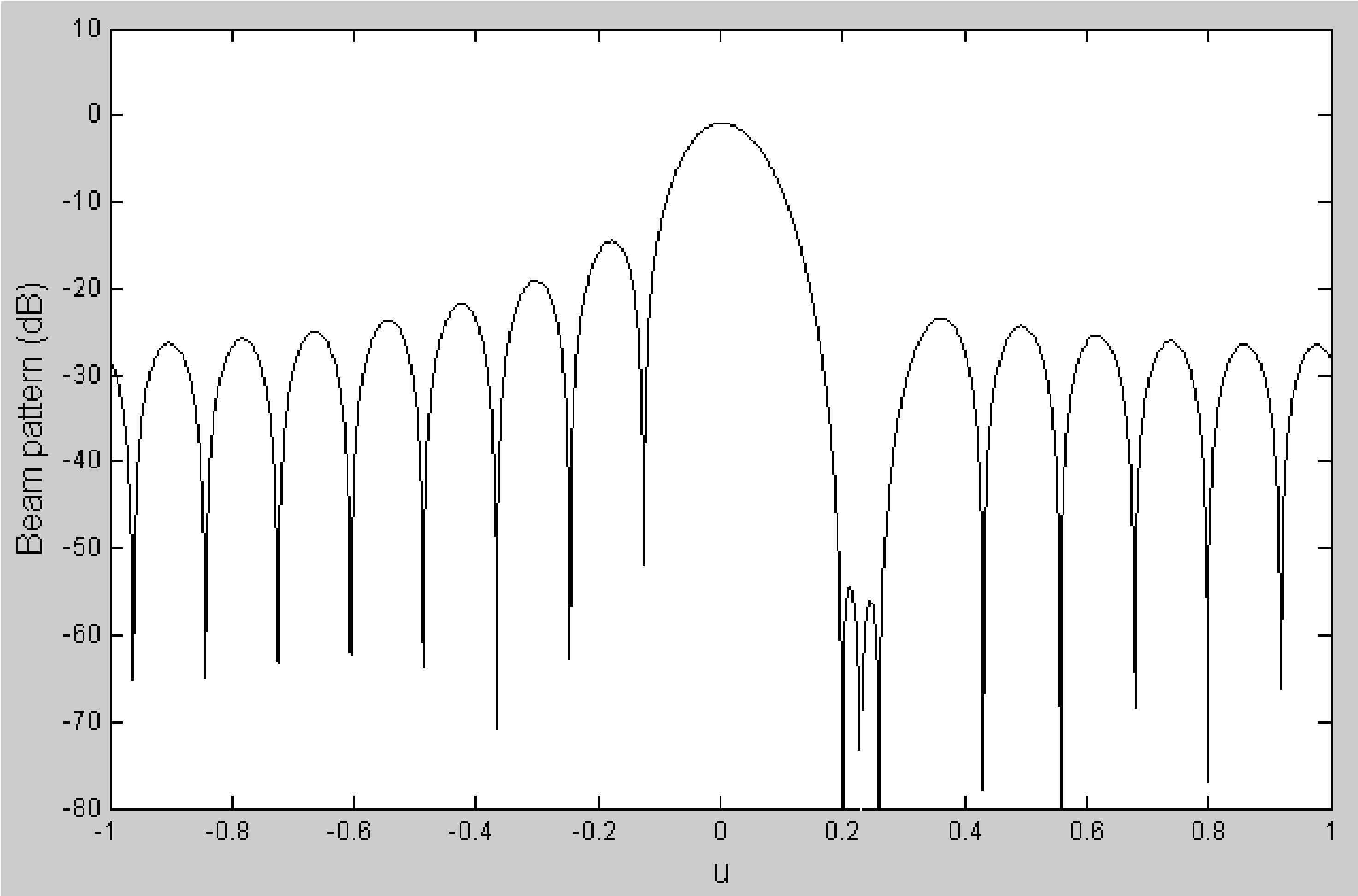
$$B_o(u) = B_d(u) - \sum_{i=1}^{M} a_i \frac{\sin(N\pi(u-u_i)/2)}{\pi(u-u_i)/2}$$

- For *N*=17 and placing nulls at

$$u_1 = 0.20,\ u_2 = 0.23,\ u_3 = 0.26$$

The pattern is as shown in next slide

Null Steering

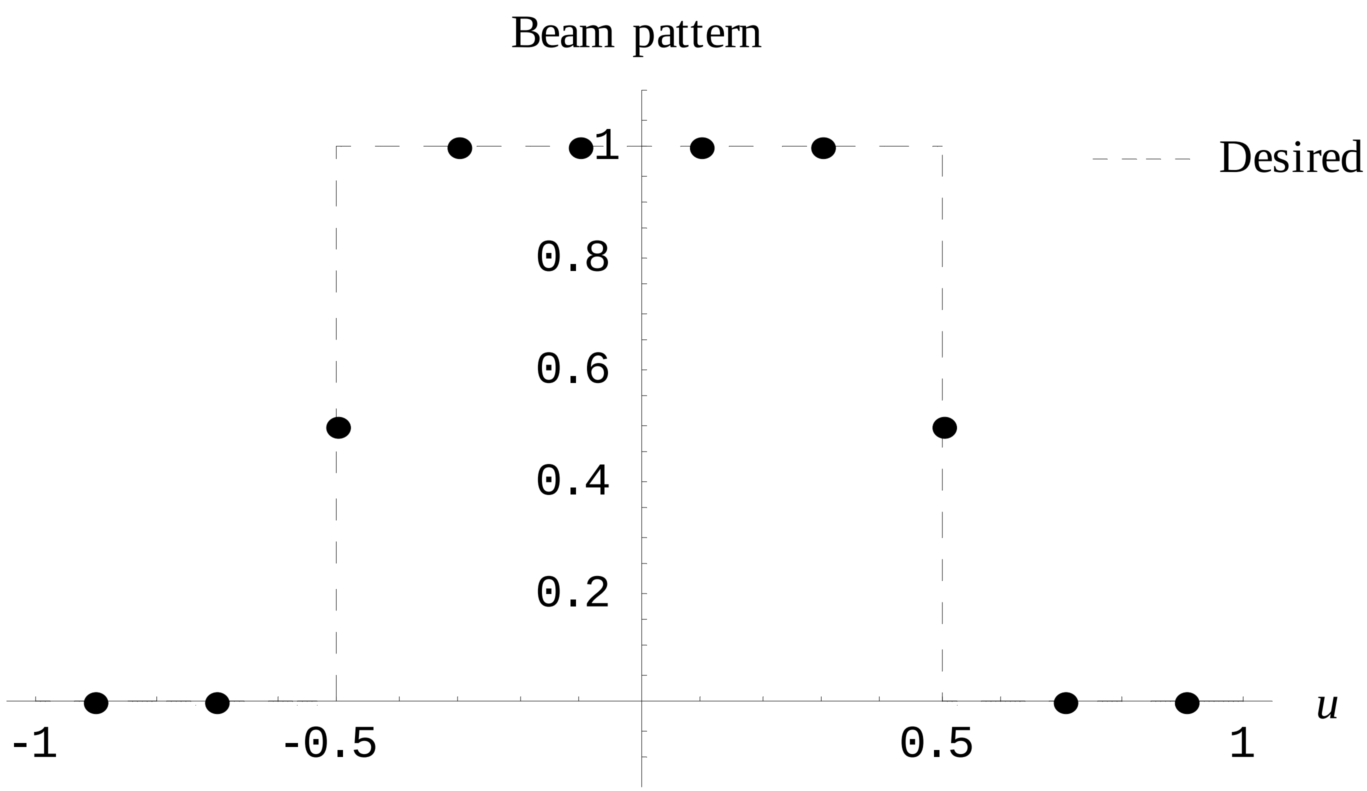


Pattern Sampling in the Wavenumber space

- The ideal beam pattern is sampled at a number of points.
- These samples are then used to reconstruct the desired pattern.
- This procedure leads to a Discrete Fourier Transform (DFT) relationship sampled points and the array weights

$$B(k) = \sum_{n=0}^{N-1} b(n) e^{jk2\pi/N} \qquad w(n) = b(n) e^{-jn\pi(N-1)/N}$$

Pattern Sampling in the Wavenumber space



Conclusions

- A modified Hamming window was presented as a better replacement for the existing Hamming window
- The effect of varying ‘a’ (in the modified Hamming) on the beampattern was discussed.
- The relationship between Z-transform and beampattern was analyzed

Conclusions

- The least-squares technique led to a Fourier series relationship.
- Woodward's pattern sampling resulted in discrete Fourier transform relationship.
- The most useful of all these techniques is the null steering method applicable to situations such as to eliminate jamming.

THANK YOU