The Selection of Weighting Functions For Linear Arrays Using Different Techniques

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17 Apr 2007
Rajarshi Shahu College of Engineering
National Conference on Electronics and Communication 2007
Pune

Abstract

Uniformly spaced linear arrays are the most elementary in array processing techniques. In this paper we consider four methods of selecting the weighting functions for uniformly-spaced linear arrays. We present a new innovation of the Hamming window, called the modified Hamming window, with a variable parameter 'a' so that the desired beampattern can be adjusted. This window is further utilized in the null-steering method.

The frequency-wavenumber response $\Upsilon(\omega, \mathbf{k})$ *at the frequency* ω *and wavenumber* $\mathbf{k}$ *is defined as the continuous domain Fourier Transform of spectral weights* w_n*. For linear arrays with equal spacing,* $\Upsilon(\omega, \mathbf{k})$ *reduces to the familiar Discrete Fourier Transform (DFT) so that the results from time domain can be used directly. However there are two important differences between array processing and time domain processing:*

> *(i) In time domain signal processing, for a given signal and noise model, the number of samples can be varied to change performance. But in array processing, we have two dimensions to take care of:* N*, the number of sensors and* K*, the number of samples that can be varied to improve the performance*
>
> *(ii) In some cases, maintaining the amplitude and phase calibration of the sensors is difficult for a number of reasons (for example, mutual coupling) and the overall array calibration changes due to changes in sensor location.*

The methods analyzed are: (i) spectral weighting using windows (ii) minimum beamwidth for a specified side-lobe level (iii) Least Squares Error Pattern Synthesis (iv) null steering

Keywords : Linear array, Hamming window, spectral weighting, null steering, least squares pattern

1 Introduction

Linear arrays are currently of great interest because of their ability to null interference and track desired signals automatically. The fundamental design problem in linear arrays is the criterion for choosing the array weights. We present a number of techniques in this paper in order to realize this. The method to choose in a practical implementation depends on several factors such as mutual coupling, array dimensions, cost per sensor and noise effects.

2 Problem Formulation

Consider the system shown in Figure 1, which shows a general narrowband beamformer. The input to each sensor is a delayed version of the input signal $f(t)$ by an amount τ_n where τ_n is the delay in $f(t)$ at the n^{th} sensor. Here N is the total number of sensors.

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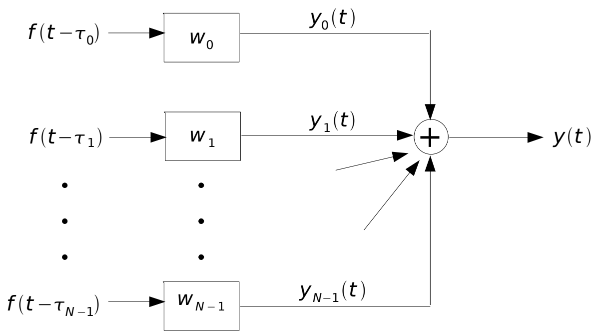

Figure 1: A narrowband beamformer

These inputs are weighted by w_n and the corresponding sensor outputs are summed to form $y(t)$. Our aim is to find the weights according to certain criteria depending on particular method to be explored in the following sections.

The inter-element distance is d and there are N elements. In general, the weights w_n are function of distance x. The azimuthal angle is ϕ.

3 Spectral Weighting Using Windows

A modified Hamming window with weights $w(n)$ is specified by:

$$w(n) = w_1(n)w_2(n) \tag{1}$$

$$\begin{aligned} w_1(n) &= 0.54 + 0.46\cos(4\pi n/N) \\ w_2(n) &= \exp(-a^2n^2/N) \end{aligned}$$

where $|n| \leq N/2$. The scaling factor is $a > 0$. For $N = 12, d = \lambda/2, a = \sqrt{3}$ the magnitude pattern is shown in Figure 2.

The beam pattern is defined by

$$B(u) = \sum_{n=-N/2}^{N/2} w(n)\exp(-j2\pi dnu/\lambda) \tag{2}$$

and $20\log(10, |B(u)|)$ gives the magnitude pattern.

Table 1 gives the comparison with other windows [1] for $N = 12, d = \lambda/2, a = \sqrt{3}$

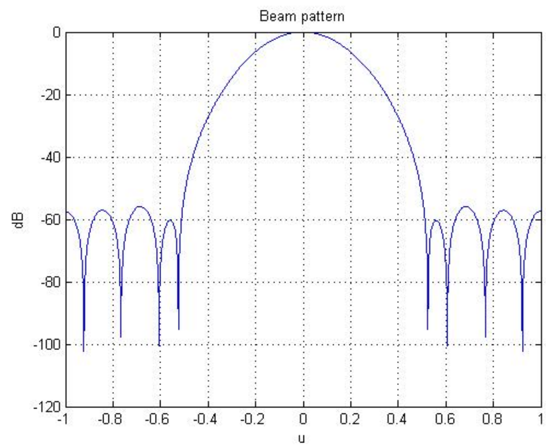

Figure 2: Beam pattern for $a = \sqrt{3}$

Weighting	First sidelobe(dB)	HPBW	FNBW
Uniform	-13.0	0.148	0.333
Hann	-31.4	0.120	0.666
Hamming	-45.0	0.109	0.668
Blackman	-57.1	0.137	0.950
Modified-Hamming	-60.3	0.138	1.050

Table 1: Comparison of windows

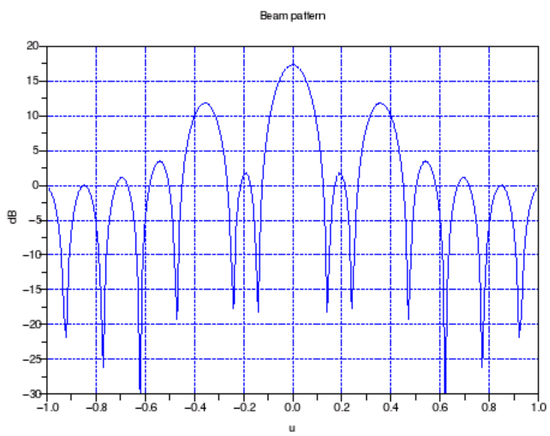

Figure 3: Beam pattern for $a = 0.5$

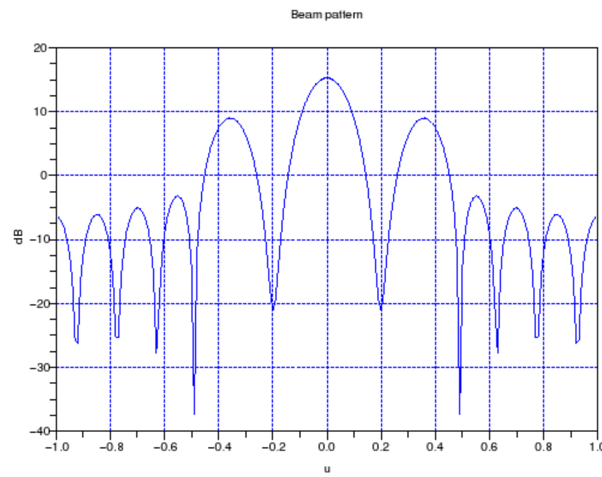

Figure 4: Beam pattern for $a = 5$

The value of a controls the sidelobe level. The peak sidelobe level is small for smaller values of a. As the value of a increases we obtain larger sidelobes.

Figure 3 shows the beampattern for lower value of $a = 0.5$ and Figure 4 shows the same for larger value of $a = 5$.

4 Minimum Beamwidth for a Specified Sidelobe Level

Let us define the ratio R as,

$$R = \frac{\text{mainlobe maximum}}{\text{sidelobe level}} \tag{3}$$

Let $T_n(x)$ denote the n^{th}–order Chebyshev polynomial of first kind in the variable x [4, 5]

We define x_0 such that

$$T_{N-1}(x_0) = R \tag{4}$$

The desired beampattern is given by [6]

$$B(\psi) = \frac{1}{R} T_{N-1}(x_0 \cos(\frac{\psi}{2})) \tag{5}$$

This method is called *Dolph-Chebyshev* method. The required weights can be obtained by [7]

$$\mathbf{w} = [\ \mathbf{V^H}(\mathbf{\Psi})\ ]^{-\mathbf{1}} \mathbf{e_1} \tag{6}$$

where $\mathbf{w}$ is $N \times 1$ weight vector, $\mathbf{V}(\mathbf{\Psi})$ is $N \times N$ array manifold matrix and $\mathbf{e_1}$is $N \times 1$ vector whose first element is 1 and the rest are zero.

For example, if $R = 15$ and $N = 8$ then $x_0 = 1.1203$. The beampattern plotted using Equation 5 is shown in Figure 5. The sidelobes are constant at 24 dB.

Taylor generalized the method for linear apertures, see [8]

5 Least Squares Error Pattern Synthesis

Let $B_d(\psi)$ be the desired beampattern, $\mathbf{w}$ be the weight vector and $\mathbf{v}(\psi)$ be the array manifold vector. We minimize the square error ξ,

$$\xi = \int_{-\pi}^{\pi} \left| B_d(\psi) - \mathbf{w^H} \mathbf{v}(\psi) \right|^2 d\psi \tag{7}$$

by taking the complex gradient with respect to $\mathbf{w}$ and equating it to zero. If N is even, the resulting beampattern can be written as a Fourier series

$$\hat{B}(\psi) = \sum_{k=-N/2}^{k=N/2} X_k w(k) e^{jk\psi} \tag{8}$$

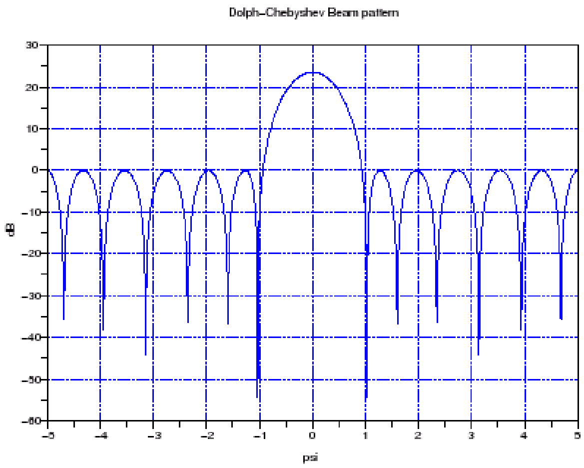

Figure 5: Beamwidth for a specified sidelobe

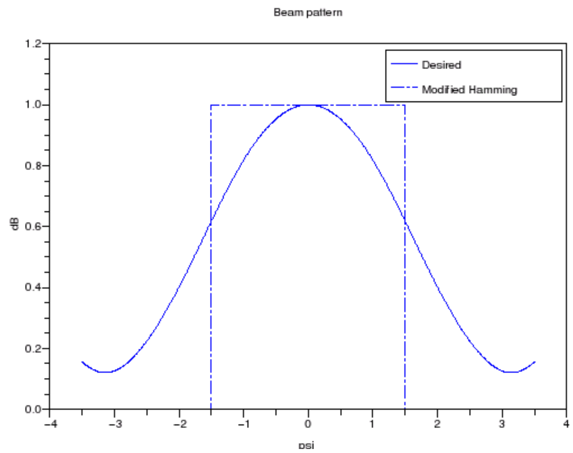

Figure 6: Beamwidth for a specified sidelobe

The Fourier coefficients are given by

$$X_k = \frac{1}{2\pi}\int_{-\pi}^{\pi} B_d(\psi)e^{-jk\psi}\,d\psi \tag{9}$$

$w(k)$ is any window discussed in section 3. Often we model $B_d(\psi)$ similar to a lowpass filter constant in $-\psi_0 \leq \psi \leq \psi_0$ and zero elsewhere.

$$X_k = \frac{\sin((k-0.5)\psi_0)}{(k-0.5)\psi_0} \tag{10}$$

For the Modified Hamming window considered in section 3, $\psi_0 = \pi/2$, $N = 12$,

$$\hat{B}(\psi) = \sum_{k=-N/2}^{k=N/2} \frac{\sin((k-0.5)\pi/2)}{(k-0.5)\pi/2} w(k)e^{jk\psi} \tag{11}$$

where $w(n)$ is given by Equation 1.

The normalized pattern is shown in Figure 6.

The use of a window function gets rid of oscillations caused by Gibbs phenomenon which would otherwise be present in a Fourier series.

6 Null Steering

In applications such as radar we intentionally want to place nulls in a particular direction. For an arbitrary array, to put a null at M points, the wavenumber $\mathbf{k_i}$ must satisfy the (zero-order) null constraint:

$$\mathbf{w^H v(k_i)} = 0, \quad i = 0, 1, 2, \ldots, M \tag{12}$$

The $N \times M$ constraint matrix $\mathbf{C}$ is defined as

$$\mathbf{C} = [\ \mathbf{v(k_1)} \vdots \mathbf{v(k_2)} \vdots \cdots \vdots \mathbf{v(k_M)}\ ] \tag{13}$$

where $\mathbf{v(k_i)}$ is the array manifold vector. Let $\mathbf{w_d}$ be the set of desired weights with the corresponding pattern $B_d(u)$. Define an auxiliary weight vector $\mathbf{a}$ as

$$\mathbf{a} = \mathbf{w^H C (C^H C)}^{\mathbf{-1}} \tag{14}$$

Performing the optimization using Lagrange's multipliers and least squares error pattern criterion as in section 5 the output pattern is

$$B_0(u) = B_d(u) - \sum_{i=1}^{M} a_i \frac{sin(N\pi(u-u_i)/2)}{\pi(u-u_i)/2} \tag{15}$$

where u_i is the null direction corresponding to $\mathbf{k_i}$. For example, taking $N = 17$ nulls are placed at $u_1 = 0.20, u_2 = 0.23, u_3 = 0.26$. We have reduced the highest sidelobe in the sector $u_1 \leq u < u_2$ to -55 dB and in the sector $u_2 \leq u < u_3$ to -56 dB (**??**). The desired beampattern was taken to be uniform weighting (ideal case).

7 Conclusions

A modified Hamming window was introduced as an alternative to existing windows. The relationship between the Z-transform and beampattern was examined. The Dolph-Chebyshev method illustrated the optimum method for minimizing the beamwidth for any given sidelobe level.

The least-squares technique led to a Fourier series whereas the Woodward sampling resulted in discrete Fourier transform. The most useful of all these techniques is the null steering method applicable to situations such as to eliminate jamming.

The above methods also exemplify the close relationship existing between time-domain signal processing and array processing.

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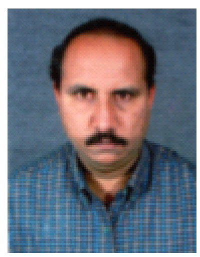

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